Fractional Dead-time Modeling for Variable Sampling to Actuation Delay Compensation in Distributed Real-Time Control Systems

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Abstract. This paper presents the study of the influence of the variable sampling to actuation delay in a distributed real-time adaptive control system and presents the results obtained by modeling the variable sampling to actuation delay as a fractional dead-time. The results show that the use of the identification model considering the variable sampling to actuation delay as fractional dead-time are better then the ones obtained with the classical model and that the improvement that can be achieved is higher for higher mean values of the delay introduced and higher variations in the delay from sample to sample.

1 Introduction

Distributed control systems are nowadays widely used in embedded applications and their field of application will continue to grow as reported in [1]. In distributed control systems the sensor, the controller and the actuator are implemented in different nodes that are connected by a network and usually several distributed controllers share the same network in order to reduce the final system cost.

The distribution of control systems introduces the following problems: sampling jitter, variable sampling to actuation delay and actuation jitter to name but a few [2]. Mainly these problems arise with the use of a network to communicate between the different controller nodes but they are also caused by the computation time inside each node. Some of these problems can be avoided by a careful choice of the implementation of the system but others cannot be avoided without over constraining the global system.

The variable sampling to actuation delay is an example of this last kind of problem. It leads to the degradation of the control performance [3]- [8] but a solution to avoid it would limit the flexibility of the global system. Therefore solutions have to be investigated that allow acceptable levels of control performance without jeopardizing the system flexibility.

This work analysis the control performance of a distributed control system with a second order plant under variable sampling to actuation delay and the effect of the use of an identification model that models the variable sampling to actuation delay as fractional dead-time.

The results show that the control performance is improved when using the model that takes into account the variable sampling to actuation delay.

2 The Distributed System

The distributed control system that will be studied is presented in figure 1.

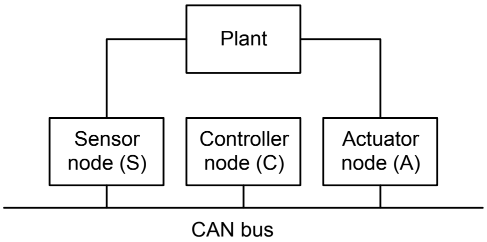

Fig. 1. Block diagram of the distributed control system

The system is composed of three nodes: sensor node, controller node and actuator node. The nodes are connected over a CAN bus. The sensor node samples the plant and sends a message with the sampled value to the controller node. The controller node computes the control signal and sends it with the actuation order to the actuator node. Finally the actuator node applies the control signal to the plant.

The plant transfer function is presented in equation 1.

$$\frac{Y(s)}{U(s)} = \frac{1.5}{s^2 + 0.5s + 1.5} \qquad 1$$

The distributed system was simulated using TrueTime, a MATLAB/Simulink based simulator for real-time control systems [9]-[11].

Each node of the system was implemented using a TrueTime Kernel block and the CAN bus was implemented using a TrueTime Network block [12].

3 The Adaptive Controller

Two models were used to identify the distributed system, one ignoring the variable sampling to actuation delay and another accounting for it. The controller used is of the pole-placement type. The adaptive controller was implemented in MATLAB inside the TrueTime Kernel block of the controller node. Details on the system identification and the control function will be given in the following sections.

3.1 System identification

The generic form of the discrete time transfer function to be modeled is given by equation 2 with the polynomials A and B given by equations 3 and 4 and T_d is the discrete dead-time.

$$G(q^{-1}) = q^{-T_d} \frac{B(q^{-1})}{A(q^{-1})} \qquad 2$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_n q^{-n} \qquad 3$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \ldots + b_m q^{-m} \qquad 4$$

The system identification is usually based on a model that ignores the variable sampling to actuation delay. The model proposed for the SISO (Single Input Single Output) system is given in this case by equations 5 and 6,

$$\frac{dx(t)}{dt} = A_1 x(t) + B_1 u(t) \qquad 5$$

$$y(t) = x(t) \qquad 6$$

The discrete model can be represented by equation 7,

$$y_1(kh+h) = \Phi_1 y(kh) + \Gamma_1 u(kh) \qquad 7$$

where

$$\Phi_1 = e^{A_1 h} \qquad 8$$

$$\Gamma_1 = \int_0^h e^{A_1 s} ds B_1 \qquad 9$$

The discrete transfer function is given by equation 10 that, for a second order system, leads to 11.

$$G_1(q) = \begin{bmatrix}1 & 0\end{bmatrix} (qI - \Phi_1)^{-1} (\Gamma_1 q^{-1}) \qquad 10$$

$$G_{m1}(q^{-1}) = \frac{b_{11} q^{-1} + b_{12} q^{-2}}{1 - a_{11} q^{-1}} \qquad 11$$

The parameters of equation 11 are in theory constant. Nevertheless when using this model on-line to model the characteristics of a system subject to variable sampling to actuation delay the parameters identified will be changing because the identifier (that doesn't account for the variable sampling to actuation delay) will "see" the system as being variable over the time.

In [13] and [14] a different model is used to identify the system, that models the variable sampling to actuation delay as fractional dead-time. The SISO (Single Input Single Output) system is given by equations 12 and 13,

$$\frac{dx(t)}{dt} = A_2 x(t) + B_2 u(t - \tau) \qquad 12$$

$$y(t) = x(t) \qquad 13$$

where τ represents the variable delay which can be considered as a dead-time. If the delay is bounded by the sampling period h, $\tau < h$, the discrete model can be represented by equation 14,

$$y(kh+h) = \Phi_2 y(kh) + \Gamma_2 u(kh) + \Gamma_3 u(kh-h) \qquad 14$$

where

$$\Phi_2 = e^{A_2 h} \qquad 15$$

$$\Gamma_2 = \int_0^{h-\tau} e^{A_2 s} ds B_2 \qquad 16$$

$$\Gamma_3 = e^{A_2(h-\tau)} \int_0^{\tau} e^{A_2 s} ds B_2 \qquad 17$$

The discrete transfer function is given by equation 18 that for a second order system leads to equation 19.

$$G_2(q) = [1 \quad 0](qI - \Phi_2)^{-1}(\Gamma_2 + \Gamma_3 q^{-1}) \qquad 18$$

$$G_{m2}(q^{-1}) = \frac{b_{21}q^{-1} + b_{22}q^{-2} + b_{23}q^{-3}}{1 - a_{21}q^{-1} - a_{22}q^{-2}} \qquad 19$$

The parameters of equation 19 vary according to the variable sampling to actuation delay introduced by the system in each control cycle are also identified on-line.

The difference between equations 11 and 19 is that equation 19 presents an extra zero, that doesn't exist in equation 11. This extra parameter will account for the effect of the variable sampling to actuation delay. The model in equation 11 will be called model 1 and model in equation 19 will be called model 2.

The discrete functions for models 1 and 2 were obtained using the parametric-model ARX (Auto-Regressive with an eXogenous signal) [13] that is appropriate to operate with a control function of the pole-placement type.

The system parameters were estimated using the least squares criterion and a recursive implementation using forgetting factor was adopted to run on-line during the simulation. The regressors for model 1 and model 2 are given in equations 20 and 21.

$$y_1(k) = \left[y_1(k-1) \quad y_1(k-2) \quad u_1(k-1) \quad u_1(k-2) \right] \qquad 20$$

$$y_2(k) = \left[y_2(k-1) \quad y_2(k-2) \quad u_2(k-1) \quad u_2(k-2) \quad u_2(k-3) \right] \qquad 21$$

3.2 The control function

The pole-placement technique was used to implement the control function. With this technique the closed-loop behaviour of the system can be specified in advance. The specification is made by the appropriate choice of the poles of the closed-loop transfer function of the system. An observer polynomial with faster dynamics is also used.

The closed-loop characteristic polynomial for the system under study was chosen having bandwidth equal to w_m=1.8 rad/s and damping factor z_m=0.7. The observer polynomial has bandwidth w_o=3.6 rad/s and damping factor z_o=0.7. The parameters of the control functions were obtained by directly solving the Diophantine's equation for each of the models. The resulting control functions are given in equations 22 and 23.

$$u_1(k) = -s_{01}y(k) - s_{11}y(k-1) - s_{21}y(k-2) - (r_{11}-1)u(k-1)\ldots$$
$$\ldots + r_{11}u(k-2) + \frac{p_{s1}}{(b_{11}+b_{12})}\left(r(k) + o_{11}r(k-1) + o_{21}r(k-2)\right) \quad 22$$
$$u_2(k) = (1-r_{22})u(k-1) + (r_{22}-r_{32})u(k-2) + r_{32}u(k-3) - s_{02}y(k)\ldots$$
$$\ldots - s_{12}y(k-1) - s_{22}y(k-2) + \frac{p_{s2}}{(b_{21}+b_{22}+b_{23})}\left(r(k) + o_{12}r(k-1) + o_{22}r(k-2)\right) \quad 23$$

4 Description of the experiments

Several tests were made with both models of the system varying the sampling to actuation delay conditions.

The sampling period (h) was chosen following the rule of thumb from [14]. This rule states that the product between the natural frequency and the sampling period should be between 0.2 and 0.6. The sampling period is 330ms, corresponding to the upper limit of the interval that is the one that allows more flexibility when using a network scheduler to schedule the network traffic in distributed systems were several controllers share the same network.

Several sampling to actuation delay conditions were tested using models 1 and 2.

The variable sampling to actuation delay was introduced in the system and follows different temporal distributions. Two of these sequences use values obtained by randomly choosing values inside the intervals [0, 0.75*h] and [0,h]. The first sequence is called rd75 and the second one rd100. The sampling to actuation delay obtained using these sequences is presented in figure 2.

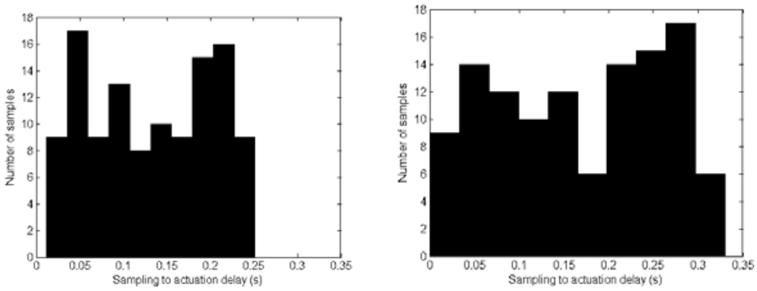

Fig. 2. Sampling to actuation delay obtained with sequences rd75 (left) and rd100 (right)

Figure 3 presents the sampling to actuation delay obtained with sequences gam and invgam. Both of these sequences are based on the gamma distribution. In the sequence invgam the values obtained with the gamma distribution are inverted inside the interval [0, h] in order to obtain values concentrated in the upper part of the interval. The gam sequence uses the gamma distribution directly and the values obtained are mainly concentrated inside the interval [0, h/2].

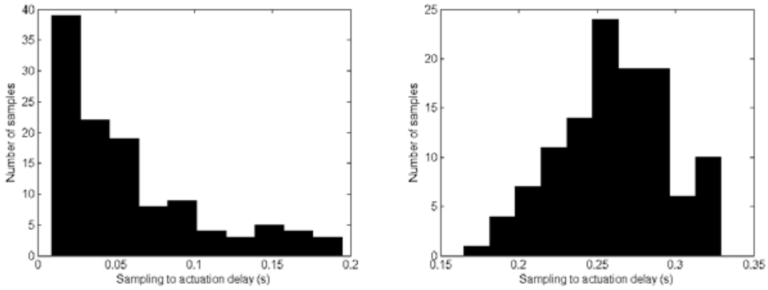

Fig. 3. Sampling to actuation delay obtained with sequences gam (left) and invgam (right)

Figure 4 presents the sampling to actuation delay obtained with sequence norm. Sequence norm was obtained using the normal distribution and the values are centered in h/2.

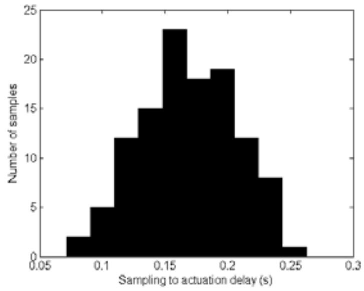

Fig. 4. Sampling to actuation delay obtained with sequences norm

Table 1 presents the mean value and the standard deviation for each of the sequences tested.

Table 1. Mean value and standard deviation for the sequences tested

Delay	Mean (ms)	Std
rd75	125	0,07
rd100	166	0,09
norm	165	0,04
gam	59	0,06
invgam	251	0,04

The invgam sequence presents the higher mean value the rd100 presents the higher standard deviation.

A reference test was made accounting only for the computing delays and the MAC (Medium Access Control) access.

The forgetting factor used for system identification is equal to 0.99.

5 Results of the experiments

The control performance was accessed by the computation of the ISE (Integral of the Squared Error). The results obtained for each test are reported in table 1.

Table 1. ISE value for h=330ms

Delay	ISE Mod. 1	ISE Mod. 2	Imp. (%)
ref.	6,24	6,24	0
rd75	7,93	7,66	16,2
rd100	8,53	8,11	18,5
norm	7,75	7,52	15,3
gam	6,63	6,57	16,7
invgam	8,33	7,95	18,4

The reference is a step signal with edges at t=15s and t=25s. The total test duration equals 40s. ISE was computed between t=13s and t=38s not accounting for the initial part of the test where the identifier is adapting to the system.

Mod. 1 refers to the test using the identification model 1 and Mod. 2 to the one using model 2. The last column in the table presents the improvement achieved using model 2 compared with the ISE increase obtained with model 1 caused by the sampling to actuation delay.

The control signals for the reference test are presented in figure 5.

Model 1 tests' are presented on the left side of the figure and model 2 tests' on the right side.

The initial part of the tests is not presented because the identifier is adapting to the system and the control signal changes inside the interval [-20, 20] could hide the details from the rest of the test.

The reference test that accounts for the computational delay and the MAC access presents the same control performance for both models as can be confirmed from figure 5.

The control signals for the sequences rd100 and invgam, corresponding to the bigger improvements in the control performance are presented in figures 6 and 7.

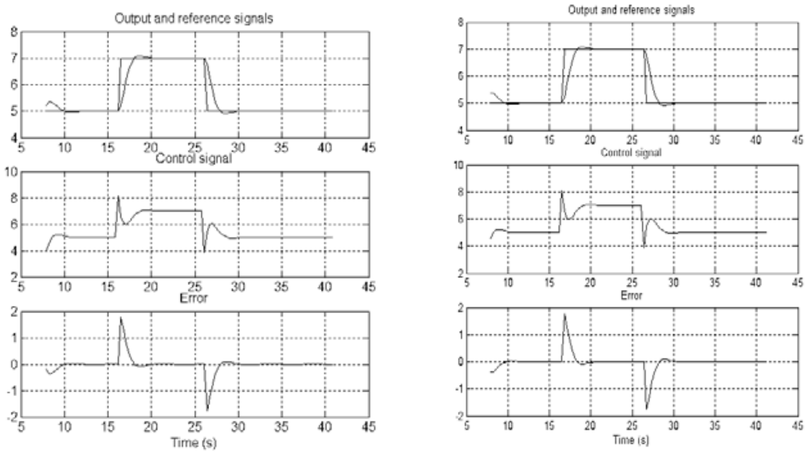

Fig. 5. Control signals for the reference test

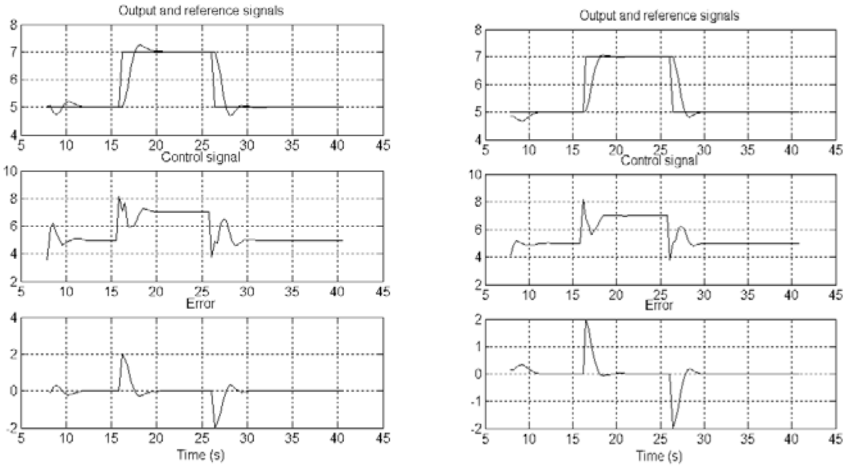

Fig. 6. Control signals for the rd100 test

The results show that model 2 allows better control performance in all the tests that present variable sampling to actuation delay.

The improvement obtained varies depending on the distribution of the sampling to actuation delay and ranges from 15% to 21%.

The control performance and the improvement that can be achieved are related to the mean value of the delay introduced and to the standard deviation, that is the variation of the delay from sample to sample.

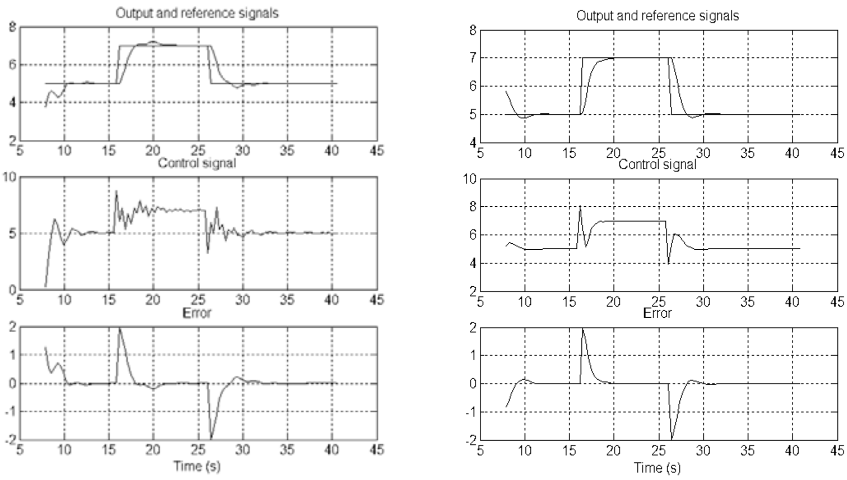

Fig. 7. Control signals for the invgam test

Better results are achieved when using the sequences that present bigger changes in the sampling to actuation delay from sample to sample (rd100) combined with higher mean value of the delay introduced (invgam). For the rd100 sequence the mean value is equal to 165 ms (h/2) but the standard deviation is the highest (0,09). For the invgam sequence the changes in the sampling to actuation delay are mainly concentrated in the interval [0.5*h h] having a mean value of 251 ms (0.76*h) and a standard deviation of 0.04.

The results obtained show that model 2 allows better results then model 1 and presents bigger improvements in the control performance when modeling systems where the variations in the sampling to actuation delay is higher from sample to sample or when the mean value of the delay introduced is high (when compared with the sampling period).

5 Conclusions

An adaptive distributed control system was tested under variable sampling to actuation delay using different models to identify the system.

Model 1 ignores the variable sampling to actuation delay effects and model 2 models it as a fractional dead-time.

The results obtained showed that model 2 allows a better control performance. The results are better when the changes in the sampling to actuation delay are bigger from sample to sample and when the mean value of the delay introduced approaches h.

This method was also applied to first order plants [15], [16] with good results. Another application of this method is presented in [17] to allow more flexibility to the network scheduler of a distributed system with several controllers sharing the same network.

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