Abstract. In this paper we review some aspects of the theory of defeasible conditionals that the late Carlos Alchourrón developed in the last years of his life. These include both philosophical intuitions and formal features of his theory. In particular, we discuss the concept of a contributory condition used by Alchourrón, his formalization of the notion of Prima facie duty and the connection between his theory of defeasible conditionals and the AGM logic of theory change.

Content areas: belief revision, defeasible conditionals, Prima facie duties, theory of conditionals.

1 Introduction

In the last years of his life Carlos Alchourrón published a series of articles on the logic of defeasible conditionals (Alchourrón, 1993, 1994, 1996). In these papers he proposed a philosophical elucidation of the notion of defeasibility and applied it to clarify deontic concepts such as that of Prima facie duty.

Alchourrón’s goal in these papers can be described as the deconstruction of the notion of defeasibility used in the AI literature, which he considers previous to the determination of the “true” logic of defeasible conditionals. In fact, as himself points out, his logic of defeasible conditionals can be seen as an axiomatization of one of the systems introduced by B. Hansson in (Hansson, 1969). So, the merits of Alchourrón’s work in this field must be evaluated taking into account its philosophical significance rather than its mathematical novelty. His analysis of the intuitive notion of defeasible conditional is made in terms of the concepts of the implicit content of a statement (assumptions) and contributory condition, the relation between which he does not problematize. The formal counterparts of these intuitive notions are given in terms of classical logic and the logic of theory change. As R.P. Loui (Loui, 1997) points out, Alchourrón wants to prove that the dark notion of defeasibility can be spelled out using clearer and safer concepts. The relation between the intuitive and formal concepts is displayed in the following diagram.
Alchourrón’s main theses on the subject can be summarized as follows:

Th1. The conditional constructions of ordinary language are often used in such a way that the antecedent $A$ together with a set of assumptions accepted in the context of utterance of the conditional, but not by itself, is a sufficient condition for the consequent $B$. In these cases $A$ is a (proper) contributory condition, i.e. a necessary condition of a sufficient condition, for $B$.

Th2. “$A$ is a sufficient condition for $B$” can be represented by “$A \Rightarrow B$”, where $\Rightarrow$ is S5 strict implication operator. The necessity $\Box$ (possibility $\Diamond$) operator of S5 must be interpreted as expressing a kind of factual necessity (possibility).

Th3. “$A$ is a contributory condition for $B$” can be represented by “$fA \Rightarrow B$”, where $fA$ is used to symbolize the joint assertion of $A$ and the set of assumptions that comes with it. “$A \succ B$” is used as a shorthand for “$fA \Rightarrow B$”.

Th4. Given the intended intuitive interpretation, the adequate set of axioms for the operator $f$ is:

\begin{align*}
(f.1) & \vdash (fA \supset A) & \text{(Expansion-$f$)} \\
(f.2) & \vdash (A \leftrightarrow B) \supset (fA \leftrightarrow fB) & \text{(Extensionality-$f$)} \\
(f.3) & \vdash (\Box A \supset \Box fA) & \text{(Limit Expansion-$f$)} \\
(f.4) & \vdash ((f(AvB) \leftrightarrow fA) \lor (f(AvB) \leftrightarrow fB) \lor (f(A \lor B) \leftrightarrow (fA \lor fB)))) & \text{(Hierarchical Ordering-$f$)}
\end{align*}

The properties of the strict implication operator $\Rightarrow$ and of the $f$ operator determine a conditional logic, the DFT system.

Th5. The operator $f$ is a special AGM revision function. AGM is the logic of theory change developed by C. Alchourrón, P. Gärdenfors and D. Makinson (Alchourrón, Gärdenfors and Makinson, 1985).

Th6. Defeasible conditionals can be identified with those conditionals in which the antecedent is a contributory condition of its consequent.

Th7. Sentences expressing *Prima facie* duties can be formalized by sentences of the form $fT \Rightarrow OB$, where $O$ is the obligation operator of the standard system of deontic logic.
The purpose of this paper is the discussion of some aspects of Alchourrón’s theory of defeasible conditionals. In Section 2, we analyze the philosophical foundations of his theory in terms of the notion of contributory condition. In Section 3, we examine the plausibility of Alchourrón’s elucidation of Ross’ notion of Prima facie duty in the frame of his deontic logic AD. In Section 4, our aim is twofold: firstly, we elucidate the relation between the DFT system of defeasible conditionals and the AGM theory and, secondly, we show that Alchourrón’s account of the revision function is compatible with the Ramsey test.

2 Von Wright’s theory of conditions and the logic of defeasible conditionals

Alchourrón borrows the concept of contributory condition, a central notion in his reconstruction of the logic of defeasible conditionals, from von Wright’s theory of conditions (von Wright, 1951).

But the definition of contributory condition used by Alchourrón — $A$ is a contributory condition for $B$ iff $A$ is a necessary condition of a sufficient condition for $B$ — trivializes that notion if no further specification is made. For, given any pair of sentences $A$ and $B$, $A$ is always a contributory condition for $B$ — because $(A \land B)$ is a sufficient condition for $B$, and $A$ is a necessary condition for $(A \land B)$.

It should be noticed that this result renders this notion of contributory condition useless, even if our interpretation of Alchourrón’s theory as expressed in Thesis 3 is challenged — i.e., even if Alchourrón is interpreted as stating only that when a sentence of the form $A > B$ is true, $A$ is a contributory condition of $B$.

In order to escape this sort of triviality the following definition, also inspired by von Wright’s characterization of this notion, may be suggested: $A$ is a contributory condition of $B$ iff there exists a finite set $\phi$ of (pairwise) totally logically independent sentences to which both $A$ and $B$ belong, such that $A$ in conjunction with $n$ sentences of $\phi$ which do not include $B$, is a sufficient condition for $B$. For two sentences to be totally logically independent any combination of truth values must be logically possible for them.

But even this more stringent definition trivializes the notion of contributory condition. To see this, take $\phi$ to be made up of $A$, $B$ and $((A \supset B) \land C)$, where $A$, $B$ and $((A \supset B) \land C)$ are pairwise totally logically independent. Then, $A$ is a contributory condition for $B$, given that $A$ in conjunction with $((A \supset B) \land C)$ entails $B$.

Furthermore, the intuitive notion of contributory condition does not seem to validate some of the principles validated by Alchourrón’s logic of defeasible conditionals. To see this, suppose that $A$ is logically possible and that, as it may well happen, the following two sentences are true:

(1) $A \land B \Rightarrow C$
(2) $A \land \neg B \Rightarrow \neg C$

The interpretation of $A$ as “$a$ is an even number”, $B$ as “$b$ is an odd number”, and $C$ as “$a + b$ is an odd number” provides us with an example of a pair of true sentences of this form. In this case $A$ is a contributory condition for $C$ and it is also a contributory condition for $\neg C$. Therefore, the following sentence is also true:

(3) $(A > C) \land (A > \neg C)$

But, when $A$ is logically possible, (3) is always false according to Alchourrón’s semantics for defeasible conditionals.
Given that (3) is not invalid according to the intuitive notion of contributory condition, the following axiom of Alchourrón’s system DFT cannot be valid for a logic of defeasible conditionals inspired by that intuition:

\[(A > (B \land C)) \equiv ((A > B) \land (A > C))\]  \hspace{1cm} \text{(Consequent distribution)}

Because, if (4) was valid, then given that (3) can be true, (5) could also be true:

\[A > (C \land \lnot C)\]

But, (5) can only be true if \(fA\) is a contradiction, which cannot be the case according to the intuitive notion of contributory condition, if \(A\) is logically possible.

So, the plausibility of thesis 6 can be questioned: the sort of defeasibility involved in the notion of contributory condition does not seem to be identifiable with the kind of defeasibility formalized by systems like DFT. In particular, it does not seem identifiable with the notion of conditionality expressed by conditionals of the kind “Normally, if \(A\) then \(B\)” and “Typically, if \(A\) then \(B\)”.

A formal notion of conditionality that seems to satisfy at least some of the properties of the intuitive notion of contributory condition can be characterized semantically as follows: \(A\) is a contributory condition of \(B\) iff the intersection of the set of the \(A\)-worlds and the set of the \(B\)-worlds is not empty. This definition does not trivialize the notion of contributory condition, and can be seen as a qualification of the intuitive notion used by Alchourrón. In fact, it is equivalent to the following definition: \(A\) is a contributory condition of \(B\) iff there is a set of \(C\)-worlds such that this set is included both in the set of the \(A\)-worlds and in the set of the \(B\)-worlds, and the set of the \(C\)-worlds is not the empty set — i.e., iff \(A\) is a necessary condition of a non-vacuous sufficient condition for \(B\).

The \(>\) connective expressing this notion of conditionality is defeasible, i.e. it satisfies neither strengthening of the antecedent nor \textit{modus ponens}. Furthermore, it does not satisfy consequent distribution and, therefore, a logically possible sentence \(A\) can be a contributory condition for both a sentence \(B\) and its negation. But, this notion is also symmetric — i.e. if \(A\) is a contributory condition of \(B\), then \(B\) is a contributory condition of \(A\). Even though this seems to be a counterintuitive property for a notion of contributory condition, we conjecture that we cannot do better if we want to find a non-trivial formal counterpart of the intuitive notion of contributory condition.

3 \textit{Prima facie} duties and defeasible conditionals

Alchourrón claims that David Ross’ notion of \textit{prima facie} obligation (Ross, 1939) can be formalized in a system of deontic logic built as an extension of DFT, his axiomatics for defeasible conditionals.

In his paper “Detachment and defeasibility in deontic logic” (Alchourrón, 1996) he uses “\textit{prima facie} duty” to denote defeasible unconditional obligations. The obligation operator of this kind is defined in his deontic system AD as a DFT conditional which has a sentence of indefeasible or actual obligation as its consequent and the constant \(\top\) as its antecedent:

\[Od(A) =_{df} \top > OA\]  \hspace{1cm} \text{(defeasible unconditional obligation)}

The operator \(O\) in this definition is the obligation operator of the standard system of deontic logic, i.e. the system characterized by the following laws and rule:

\[A1. \top \vdash O(\top)\]
A2. \( \vdash O(A \land B) \equiv O(A) \land O(B) \)

A3. \( \vdash \neg O(\bot) \)

R1. If \( \vdash A \equiv B \), then \( \vdash O(A) \equiv O(B) \)

As Alchourrón points out, this operator \( Od \) also satisfies the principles of the standard system of deontic logic. It follows from the counterparts of A2 and A3 for \( Od \) that in Alchourrón’s system we cannot have conflicting \textit{prima facie} obligations, i.e.

\[ \vdash \neg (Od(A) \land Od(\neg A)) \]

On the other hand, Ross makes it quite clear that, even though actual obligations cannot conflict, we \textit{can} have conflicting \textit{prima facie} obligations:

“It is the overlooking of the distinction between obligations and responsibilities, between actual obligatoriness and the tendency to be obligatory, that leads to the apparent problem of conflict of duties, and it is by drawing the distinction that we solve the problem, or rather show it to be nonexistent. For while an act may well be \textit{prima facie} obligatory in respect of one character and \textit{prima facie} forbidden in virtue of another, it becomes obligatory or forbidden only in virtue of the totality of its ethically relevant characteristics.” (Ross, 1939, p.86)

This point may also be seen from a semantic angle. Alchourrón’s formal system can be described, using Brian Chellas’ words, as one in which “associated with each possible world ( . . . ) there is a single class of deontic alternatives”; but, when we are dealing with \textit{prima facie} duties we are faced with a “plurality of competing grounds of obligation” (Chellas, 1974) Therefore, Alchourrón’s characterization of \textit{prima facie} duties in terms of the notion of obligation of the standard system of deontic logic does not seem to be at all adequate, if we are looking for a formalization of Ross’ notion. In fact, it is incompatible with some basic features of this notion.

The \textit{prima facie} duties formalized by Alchourrón can be characterized as obligations that hold in normal circumstances, but can be defeated in abnormal ones. On the other hand, Ross’ notion of \textit{prima facie} duty tries to reflect the fact that we evaluate an act according to a plurality of moral principles, some of which may recommend it while others may disrecommend it. Therefore, a conflict of \textit{prima facie} obligations is not only possible but also not uncommon according to Ross’ theory.

The rejection of A2 as a valid principle for \textit{prima facie} obligations (in Ross’ sense) seems to be a necessary step towards an adequate logic for this kind of duties. This is in accordance with Ross’ theory: the fact that we may be under two contradictory \textit{prima facie} obligations does not mean that we are under a \textit{prima facie} obligation to perform some self-contradictory — and therefore logically impossible — act. Furthermore, because we are dealing here with \textit{prima facie} duties the conflicts of obligations that this logic would allow are not genuine conflicts: they do not face us with \textit{bona fide} moral dilemmas, i.e. situations in which it is impossible for the agent to do what (s) he ought to do.

Another point in which Alchourrón’s theory departs from Ross’ concerns the relation between \textit{prima facie} and actual duties. In Ross’ theory actual obligations are also \textit{prima facie} ones:

“ . . . we are \textit{not} obliged to do that which is only \textit{prima facie} obligatory.
We are only bound to do that act whose \textit{prima facie} obligatoriness in those respects in which it is \textit{prima facie} obligatory most outweighs its \textit{prima facie}
disobligatoriness in those respects in which it is \textit{prima facie} disobligatory."  
\cite[1939, p. 85]{Ross}

On the other hand, Alchourrón points out that the following principles are not valid in his system AD:

\[
\vdash OA \supset Od(A) \\
\vdash Od(A) \supset O(A)
\]

Therefore, in AD \textit{prima facie} duties are logically independent of actual duties. This is as it should be, given Alchourrón’s interpretation of \textit{prima facie} obligations. But, if we want to formalize Ross’ notion of \textit{prima facie} duty in a system of deontic logic such a system must necessarily include the first of those two principles.

4 Alchourrón’s defeasible conditionals, AGM revision and the Ramsey Test

The definition of a defeasible conditional \(A > B\) in his system DFT is the S5-strict implication \(fA \Rightarrow B\), where \(f\) is a revision function. We begin by determining the kind of revision defined by the \(f\) function, and its connection with the AGM model.

Let us recall that the AGM formalism accounts for three types of change: expansion, revision and contraction, which are characterized by specific postulates. In particular, for our purposes we are interested in those for revision.

\begin{align*}
K^*1) & \quad K_A^* \text{ is a theory.} \\
K^*2) & \quad A \in K_A^* . \\
K^*3) & \quad K \ast A \subseteq K_A^* . \\
K^*4) & \quad \text{If } \neg A \not\in K \text{ then } K_A^* \subseteq K_A^* . \\
K^*5) & \quad K_A^* = \text{Cn}(\bot) \iff \text{Cn}(\neg A) = \text{Cn}(\emptyset) \\
K^*6) & \quad \text{If } \text{Cn}(A) = \text{Cn}(B) \text{ then } K_A^* = K_B^* . \\
K^*7) & \quad K_A^{A \land B} \subseteq (K_A^*)+B . \\
K^*8) & \quad \text{If } \neg B \not\in K_A^* \text{ then } (K_A^*)+B \subseteq K_A^{A \land B} .
\end{align*}

The starting point is a theory \(K\), a set of language sentences closed under a logical consequence relation which contains at least the classical propositional one. Intuitively, postulate K*1 indicates that the result of revising a theory by a language sentence \(A\) is another theory \(K_A^*\), which by K*2 must include sentence \(A\).

Postulate K*3 states that the theory resulting from revision by a sentence \(A\) is extensionally smaller than or equal to the result of adding or expanding under closure theory \(K\) with sentence \(A\).

Postulate K*4 together with the previous one affirms that if \(K\) does not contain the negation of \(A\) then revising \(K\) by \(A\) is equal to expanding \(K\) with \(A\). K*5 guarantees that the result of any revision is always a consistent theory, except for the case the input sentence is itself a contradiction.

Postulate K*6 says that if two sentences \(A\) and \(B\) have identical consequences then the revision of \(K\) by \(A\) is logically equivalent to the revision of \(K\) by \(B\).

Postulate K*7 indicates that the revision of \(K\) by a conjunctive molecular sentence is extensionally smaller than or equal to the theory resulting of first revising \(K\) by one of the conjuncts and then expanding by the other.

Finally, postulate K*8 tells that if a sentence \(B\) is not determined in the revision of \(K\) by \(A\) then the revision of \(K\) by the conjunction \(A \land B\) is precisely the same as the theory that results form revising by \(A\) and then expanding by \(B\).
Let us remark that while the eight revision postulates describe properties of $K^*A$, the theory resulting from revision, only postulates $K^3$ and $K^4$ refer to theory $K$ in isolation (consider that the expansion operation can be spelled out as set theoretical addition and closure under $\text{Cn}$). We will refer back to this observation when we discuss which postulates are satisfied by the function $f$ of DFT.

In contrast with expansion, the revision function is non-monotonic in the sense that if a proposition $A$ is implied by a proposition $B$ then the theory resulting from revision by $A$ is not necessarily included in the theory resulting from revision by $B$. Formally,

$$\text{If } \text{Cn}(A) \subseteq \text{Cn}(B) \text{ then, not necessarily } \text{Cn}(K_A^*) \subseteq \text{Cn}(K_B^*)$$

4.1 The connection between AGM and DFT

Alchourrón’s contribution to the theory of defeasible conditionals has been to marry the concepts of revision and defeasible conditional in his modal system DFT using a revision function at the language level. Thus, he arrived at a new semantics for defeasible conditionals in terms of revision.

In his articles on the subject, Alchourrón refers to the function $f$ present in his definition of a defeasible conditional as a special revision function. He indicates that $fA$ must be understood as an expansion of the conceptual content of $A$, such that $fA$ reads as “$A$ and the presuppositions for $A$”. As stated in Th 4 above, this function is characterized by four axioms in the logical system DFT, f1–f4.

Alchourrón’s main contribution is not the introduction of the function $f$ at the language level, path already investigated by Stalnaker and Åquist, but its interpretation as a kind of revision function. However, this interpretation is not entirely immediate. In the formal presentation $f$ is a function of a single propositional argument, say $A$, and returns another proposition, the conjunction of $A$ and its presuppositions. However, according to the AGM paradigm, revision functions possess two arguments the theory $K$ to be revised and the input sentence $A$, and return a theory $K^*A$. As far as Alchourrón’s formalization goes, he gives no counterpart for the original theory $K$ and the input $A$; nevertheless, he dubs $f$ as a revision function in DFT.

The axioms for $f$ in DFT can be shown to be in correspondence with the AGM postulates for revision via the identification $fA$ as $K^*A$, i.e. $fA$ is the revision of an implicit theory $K$ by sentence $A$. The $f$-expansion axiom (f1) indicates that $A$ is implied by $fA$, that is, $A$ is among its own presuppositions. This is in correspondence with $K^2$. The extensionality-$f$ (f2) obviously corresponds to $K^6$ in the sense that logically equivalent sentences give rise to logically equivalent revised theories. Limit-expansion-$f$ (f3) is in correspondence with $K^5$ stating that whenever a sentence $A$ is self consistent then the theory arising from revision by $A$ is consistent too. Finally Hierarchical-ordering-$f$ (f4) is in exact correspondence with the disjunctive factoring property of AGM, which is a consequence of all the AGM postulates except $K^3$ and $K^4$.

Based on the association of $fA$ with $K^*A$, we have an identification of the axioms for $f$ and the postulates for revision except for postulates $K^1$, $K^3$ and $K^4$. Let us recall that postulate $K^1$ affirms that the result of a revision is always a theory; hence, postulate $K^1$ could be accounted for by the interpretation of $fA$ as the finite axiomatization of the theory resulting from revision by $A$. However, the absence of counterpart axioms for $K^3$ and $K^4$ deserves some reflection. $K^3$ indicates an upper limit of the revision function: the revision of a theory by the new input is always
included in the result of the expansion of the original theory with the given input. K∗3 together with K∗4 sanction that when the input does not contradict what is already in the original theory, then revision is precisely an expansion. These two postulates entail that expansion is a limiting case of revision.¹

The counterparts for K∗3 and K∗4 are derivable from the DFT axioms under a limiting assumption: when K is taken as f⊤, i.e. when K is the result of applying the revision function f to the truth constant ⊤. This assumption entails the consistency of the theory K and counts as a sufficient reason for calling f a special revision function, a less restrictive one than AGM revision.

We now turn to a different interpretation of the f operator of DFT not present in Alchourrón’s work, but motivated by his own elucidation of the f function. He explains that fA (usually) means a stronger proposition than A, a proposition that is the conceptual expansion of the proposition expressed by A (Alchourrón 1993). Therefore, we propose to interpret the f function as a particular case of the standard AGM expansion function. Under this interpretation fA corresponds to the expansion of A with its own presuppositions A₁, A₂, . . . , Aₙ, that is, fA = A ∧ A₁ ∧ A₂ ∧ · · · ∧ Aₙ.² This idea can be formalized assuming the existence of another operator g that, given a sentence, returns its presuppositions, fA = A ∧ gA, such that g must validate some minimal requirements in order for f to comply with the conditions f₁–f₄:

(g0) Function g must preserve consistency in the sense that if A is consistent then so is gA.

(g1) For any consistent proposition A, gA should not entail the negation of A.

(g2) Logically equivalent sentences have logically equivalent presuppositions.

(g3) The presuppositions of a disjunction are just the presuppositions of one of the disjuncts or the presuppositions of the conjunction. This is,

\[ g(A ∨ B) = \{gA \text{ or } gB \text{ or } g(A ∧ B)\} \]

Thus the operator f can be regarded as an expansion operator such that Cn(fA) = Cn(A ∪ gA) = A + gA, where + is the standard AGM expansion function. Curiously, the minimal requirements for function g are precisely the minimal requirements for an AGM contraction function, which is highly suggestive of the idea that the presuppositions for A are obtainable as a mirror image of the contraction by the negation of A.

We believe our interpretation is particularly appealing because it relates defeasible conditionals with the simplest AGM operation, expansion, which is defined in terms of set theory and logical closure only.

4.2 Alchourrón’s conditional logic and Ramsey test

One of the main features of the conditional logics developed in the last decade is that they include Ramsey’s proposal for the evaluation of conditionals. According to Ram-

¹The interpretation of f as a revision operator is in intimate relation with the work of Gärdenfors and Makinson (1991, 1994). Both the non-monotonic consequence relation and Alchourron’s conditional logic were explained in terms of a special revision function that satisfies all the AGM revision postulates except for K∗3 and K∗4. They also share the necessary assumption of an implicit theory K, which is of an existential nature. It can be proved that the properties of the non-monotonic consequence relation are in correspondence with those of Alchourrón’s conditional connective (with the proviso that the former is extralinguistic while the latter is not).

²Alchourrón has in mind a finite set of presuppositions for any given sentence. However, his construction is compatible with an infinite set of presuppositions with fA a finite axiomatization for such a set.
sey: “[I]f two people are arguing ‘If p will q?’ and both in doubt as to p they are adding p hypothetically to their stock of knowledge and are arguing on that basis about q” (Ramsey, 1931).

R. Stalnaker reformulated Ramsey’s suggestion in terms of the possible world semantics in order to determine truth conditions for conditionals: “First, add the antecedent (hypothetically) to your stock of beliefs; second, make whatever adjustments are required to maintain consistency (without modifying the hypothetical beliefs in the antecedent); finally, consider whether or not the consequent is then true”. Stalnaker’s semantics has been widely influential lately and his reformulation of Ramsey’s ideas is now commonly known as the Ramsey Test. Given a corpus of conditional assertions K and a revision function ∗ the Ramsey Test states that a conditional A > B belongs to K if and only if B belongs to the revision ∗ of K by A. Formally,

\[(\text{Ramsey test}) \quad A > B \in K \text{ if and only if } B \in K_a^*\]

Perhaps it would be more natural to interpret the Ramsey test epistemically, i.e. as a belief or rational acceptability condition. In modern studies of belief dynamics, which have been developed by P. Gärdenfors, Ramsey Test takes the place of “some method of revising states of belief” for conditional sentences which can be formulated “(A)ccept a proposition of the form ‘If A then C’ in a state of belief K if and only if the minimal change of K needed to accept A also requires accepting C” (Gärdenfors, 1986).

In relative recent times some philosophers have become interested in revision at quite an abstract level, seeking general methods for revising bodies of information and general constraints on the rationality of these methods. P. Gärdenfors was one of the first to introduce the notion of belief revision models in order to study the formal properties of belief revision. These models together with Ramsey Test, provide a simple interpretation for conditionals, giving as a result a natural epistemic semantics for conditionals.

Strangely enough, it was also Gärdenfors who introduced one of the most bewildering results by proving that Ramsey Test is incompatible with what seemed to be reasonable demands on minimal systems of belief revision. This is known as Gärdenfors’ Impossibility Theorem, which states that the Ramsey Test is incompatible with the AGM theory revision. Let’s illustrate Gärdenfors’ result. Consider any set K of sentences containing at least the following two conditionals A > B and (A ∧ C) > ¬B, but such that neither A, B or C (nor their negations) belong to K. Let ∗ be a revision function such that it validates all AGM revision postulates, and suppose we have to revise K by the conjunction A ∧ C. In particular, let’s take K as Cn(\{A > B, A ∧ C > ¬B\}).

By K∗2 the result of the revision operation is a new theory K∗A ∧ C that contains A and also contains C; formally, (A ∧ C) ∈ K∗A ∧ C. Since AGM requires that the revised theory be consistent then the negation of (A ∧ C) can not be in K; that is, ¬(A ∧ C) \∉ K.

By postulates K∗3 and K∗4, K∗A ∧ C is precisely the expansion of K by (A ∧ C); in symbols,

\[(6) \quad K_{A \land C}^* = K_{A \land C} +\]

Then, by the definition of the AGM expansion function,

\[(7) \quad K_{A \land C} + = Cn(K \cup \{A \land C\}) = Cn(Cn(\{A > B, A \land C > ¬B\}) \cup A \land C)\]

Let’s see now that if we accept the Ramsey Test we obtain that both B and ¬B belong to K∗A ∧ C arriving to an inconsistent theory, which contradicts K∗5.
By (4.2) and (4.2) $A > B \in K_{A \land C}^*$ and $A \in K_{A \land C}^*$. Then according to the left to right conditional in the above definition of the Ramsey Test,

$$B \in K_{A \land C}^*$$

On the other hand, by (4.2) and (4.2) $A \land C > \neg B \in K_{A \land C}^*$ and $(A \land C) \in K^*1_{A \land C}$. Hence, again by the left to right conditional of the Ramsey Test we obtain

$$\neg B \in K_{A \land C}^*$$

Several escape routes from Gärdenfors’ Impossibility Theorem have been investigated, each debilitates or abandons an intuitive demand. In particular there is Rott’s argumentation in favor of accepting both the Ramsey Test and the inclusion of conditional sentences into belief sets but renouncing to the AGM requirement that expansions preserve the original contents of a conditional theory. Rott has argued that “[e]xpansions are not the right method to ‘add’ new sentences if the underlying language contains conditionals which are interpreted by the Ramsey test” (Rot89) which is equivalent to the rejection of the AGM postulate $K^*4$ for conditional theories.\textsuperscript{3}

The Impossibility theorem confronts us with the incompatibility of apparently intuitive properties of the revision function over conditional theories. A natural question that became a recurrent open issue in personal discussions with Alchourrón has been whether his defeasible conditionals and the notion of revision in DFT are compatible with the Ramsey Test. Alchourrón never made this point explicit and one of the main difficulties lay in the adequate formulation of the problem. Åquist’s definition for conditionals allows us to evade the question, given that it substitutes the Ramsey Test as an evaluation process for conditionals. The question of whether DFT is compatible with the Ramsey test becomes significant given that a corpus of conditional assertions seems to be the way to represent normative propositions in DFT (free of purely factual statements). Suppose $K$ is a set of conditional assertions in DFT, closed under logical consequence. Then, a sentence belongs to $K$ if and only if it is derivable from $K$:

$$A > B \in K \quad \text{if and only if} \quad K \vdash_{DFT} A > B$$

Now, by its definition in DFT $K \vdash_{DFT} A > B$ is equivalent to $K \vdash_{DFT} fA \Rightarrow B$. Via the identification of $fA$ as $K_A^*$, $K \vdash_{DFT} fA \Rightarrow B$ is in mutual correspondence with $B \in K_A^*$. We have actually depicted a formulation of the Ramsey Test:

$$K \vdash_{DFT} A > B \quad \text{if and only if} \quad B \in K_A^*, \text{ where } * \text{ is the revision function induced by } f.$$ 

Does Gärdenfors Impossibility result arise in this setting? One defining feature of DFT is that the interpretation of the $f$ as a revision operator satisfies all of AGM revision postulates except for $K^*3$ and $K^*4$. According to Rott’s elucidation, lack of these postulates is a sufficient condition to avoid Gärdenfors Impossibility result.

\textsuperscript{3}I. Levy has proposed the “stratified” version of Ramsey Test. According to him conditional sentences are not “objects of belief which bear truth values and are subject to appraisal with respect to epistemic possibility and probability” (Levi, 1988). Belief states have been identified with sets of sentences that must not contain modal or conditional sentences. Another line of work suggests several weakenings of the Ramsey Test which would give a reasonable interpretation of conditional sentences that also avoids the impossibility result (Rott, 1986; Gärdenfors, 1986; Lindström and Rabinowicz, 1992). It should be noted that other logics of Theory Change, namely Katsumo-Mendelzon’s Update, are compatible with the Ramsey Test
We actually conclude the compatibility of Ramsey Test with the conditionals and the revision function of DFT.

The following graph, in which * is Alchourrón’s revision function, summarizes the above observations:

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Áqvist’s definition

fA ⇒ B

A > B

The Ramsey Test

B ∈ K_A

Alchourrón’s/Rott’s revision operator
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Thus, Alchourrón’s theory of conditionals suggests how to relate two approaches to the semantics of conditional logics usually considered incompatible in the literature: the possible worlds (modal, ontic) approach, and the purely epistemic which gives acceptability conditions in terms of the Ramsey Test.

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